# Graphical representation of functions

To represent graphically a function, we have to calculate its 'main elements', following these steps:

1.- Domain and range

2.- Symmetry and periodicity

3.- Points of intersection on **x** and **y** axis

4.- First derivative study

5.- Second derivative study

6.- Asymptotic lines

7.- Table of function values

Depending on the function features (domain, symmetries, growth, etc…) not always will be necessary to follow all the steps.

## 1.- *Domain and range*.

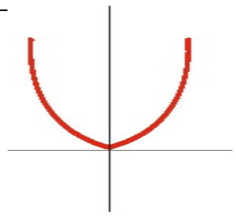
The domain of a function are all the values that make the function to exist, i.e. that the function might be calculated $Dom f(x) = \{x \in \Re \quad | \quad \exists\ f(x)\ \}$. The existence or not of the function will depend on its nature (square, quotient, logarithm…)

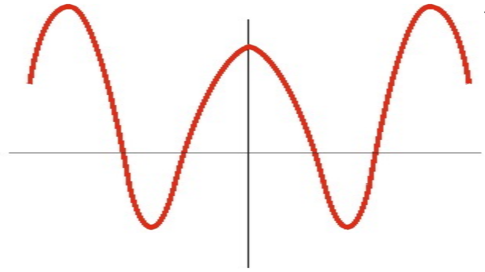
The range of a function are all the values the function can take.

## 2.- *Symmetry*.

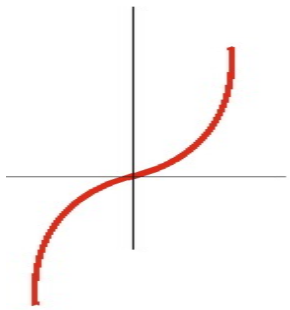
We are going to study when the function is symmetrical with respect to the y-axis (in this case, the function is called an *even function*) and when is symmetrical with respect the origin (and then it is called an *odd function)*

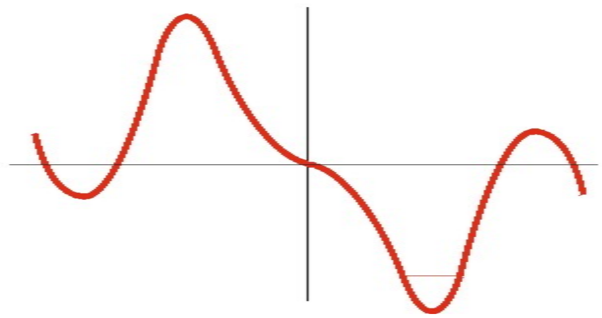
*) If $f(-x) = f(x)$, then it is an *even function.* This means that if we bend the graph following the y-axis, the negative and positive part of x-axis would match. Here you have two examples of *even functions*:





*) If $f(-x) = -f(x)$, the function is an *odd* one (symmetrical with respect to the origin). The following functions are odd:





ⓘ Remark that a function could be neither *EVEN* nor *ODD*

When required, we will also study its periodicity. A function is said to be periodic if, for some nonzero constant P, we have $f(x+P) = f(x)$

## 3.- ***Points of intersection on x and y axis***.

Intersection on x-axis: Set $y = 0$ and we solve the equation.

Intersection on y-axis: Set $x = 0$ and we replace this value in the function to calculate $y$

## 4.- ***First derivative study***.

In this section, function growth is studied, as well as relative extreme values of the function (maximum and minimum) .

*) The function *increases* in the set of points in which $f'(x) > 0$ and we will write $f \uparrow$

*) The function *decreases* in the set of points in which $f'(x) < 0$ and we will write $f \downarrow$

*) The *relative extreme points* of a function could be determined by calculating the roots of the first derivative of the function $f'(x) = 0$

We will see if these points are maximum or minimum with the second derivative of the function. (We can also study it with the first derivative and the growth, providing the function is continuous)

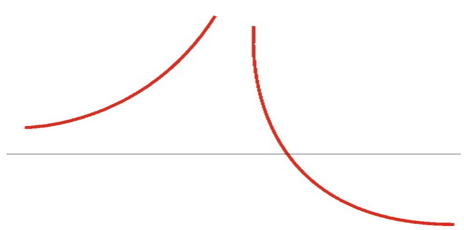
## 5.- *Second derivative study*

We calculate the second derivative (the derivative of the first derivative) and study what it happens with the set of points in which $f'(x) = 0$. We will have this situation:

If $f''(x_0) > 0$, $x_0$ is a minimum

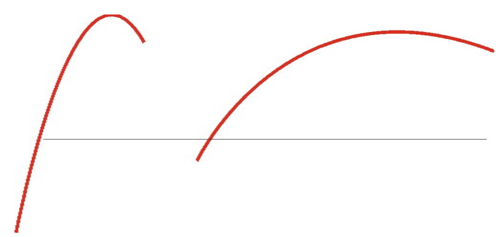
If $f''(x_0) < 0$ $x_0$ is a maximum.

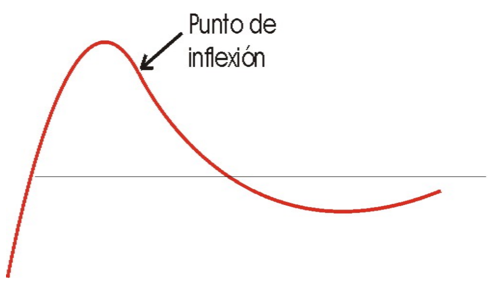
The second derivative also studies the concavity of the function, and so



*) The function will be *concave up* in $A$, if $f''(x) > 0$ in $A$

*) The function will be convave down in $A$ if $f''(x) < 0$ in $A$






There are often points at which the graph changes from being concave up to concave down, or vice versa. These points are called **inflection points**. It's clear that in these points $f''(x) = 0$ or does not exist.

## 6.- *Asymptotes*

For curves given by the graph of a function $y = f(x)$, there are potentially three kinds of asymptotes: *horizontal*, *vertical* and *oblique* asymptotes.

*) Vertical asymptotes are vertical lines near which the function grows without bound

We will say that the straight line $x = a$ is a *vertical asymptote* when:

$$\lim_{x \to a} f(x) = \pm\infty$$

*) Horizontal asymptotes are horizontal lines that the graph of the function approaches as $x$ tends to $+\infty$ or $-\infty$. We will say that the straight line $y = b$ is a *horizontal asymptote* when:

$$\lim_{x \to \pm\infty} f(x) = b$$

*) The straight line $y = mx + n \quad (m \neq 0)$ will be an *oblique asymptote* when:

$$\lim_{x \to \infty} (f(x) - mx - n) = 0 \text{, siendo} \quad m = \lim_{x \to \infty} \frac{f(x)}{x} \text{ ,} \quad \text{y} \quad n = \lim_{x \to \infty} (f(x) - mx)$$

## *7.- Table of function values*

A table of values will help you to evaluate and graph the function.